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Assignment 6

CS 453

1. We can prove that for any k≥ 3, if a tree T has fewer than k leaves, then the max degree among the vertices of T must satisfy For n = number of vertices, If a tree T has fewer than k leaves, then and is the number of leaves in the T.

Let for all j ≥ k, . This means n = . Then This means that the total degree with vertices with degrees j ≥ k doesn’t exist. Therefore, if a tree T has fewer than k leaves, then the max degree among the vertices of T must satisfy

* 1. Since r is the root, is the length of the unique u-v path. If r is on the unique u, v path, then r is an ancestor of v. Therefore, Since r is root,
  2. If , then . Since both and passes through , must be on the unique -path.
  3. If D(u,v) = 2H, then it means u and v have the maximum of levels of tree. It means they are both the end of the tree. Therefore, they are leaves and they must be non-parents.
  4. If must be two leaves with the maximum of levels of vertices. If either of is a parent, then one of they have at least one child and can’t has level of less than H. Therefore, So if then u and v must be non-parents.
  5. If b is the number of parents in the tree, then for each parent node in this tree, there are q edges and q children nodes connect to this node. Each children node, if they are not parent, have no edge connect to them. Therefore, there are edges in this tree, for n leaf nodes.
  6. For q-ary tree and b parents, there are non-parents node. Therefore, there are vertices.
  7. For q-ary tree and b parents, there are

1. There are
   1. The height of the T:
      1. Lower bound: 1
      2. Upper bound:
   2. The height of the saturated tree T:
      1. Lower bound:
      2. Upper bound: